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Fast algorithms to evaluate the approximate solution of time dependent problems were developed by taking advantage of the sparse wavelet representation of finite difference operators and using only part of the representation to compute the local solution. For example, we can evaluate the solutions at a point to parabolic equations with variable coefficients in $O(\log^4 N)$ operations when the equation has time independent coefficients. For time dependent coefficients; the complexity is $O(N \log^3 N)$. Additionally, high resolution numerical methods for the high frequency asymptotic expansion to electromagnetic propagation and scattering codes were developed. This replaces ray tracing by a direct solution to the eikonal equation. Moreover, we developed and solved generalized eikonal equations for diffraction phenomena.



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We developed fast algorithms to evaluate the solution of time dependent numerical methods for time dependent partial differential equations (PDEs) using multiscale analysis.

In [1] we developed very fast algorithms to evaluate the approximate solution to parabolic equations at one point without calculating in the whole space time domain. This was achieved by taking advantage of the sparse wavelet representation of finite difference approximations (which are analogous to those of Green's function for the operator) and using only parts of the representation to compute the local solution. The complexity for solving at a given point is only $O(\log^4 N)$ when the equation has time independent solutions. When the coefficients do depend on time, the complexity is $O(N \log^3 N)$.

Earlier, [2], we developed an algorithm which gives the solution with $O(N^d \log(N))$ complexity for linear hyperbolic equations in one space dimension and linear parabolic equation in $d$ dimensions.

In a parallel effort we developed high resolution numerical methods for the high frequency asymptotic expansions to electromagnetic propagation and scattering [3]. This replaces ray tracing by a direct solution to the eikonal equation using new numerical methods for Hamilton-Jacobi equations and conservation laws developed here.

Additionally we developed and solved generalized eikonal equations for diffraction phenomena [4]. We were able to correct for boundary effects using a new perturbed geometric optics system.

Finally, we began a program in which the eikonal equation was generalized to accomodate solutions containing many phases. New equations were developed which are based on the same high frequency approximation of the scalar wave equation as the eikonal and transport equations.

## References

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